Advances in Theoretical Finance

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Contents

1 Introduction

The theoretical foundations of finance evolve continuously. This survey summarizes key developments in finance over the years. It also presents a glimpse into the future of finance by reviewing promising areas of research.

Twentieth century finance literature includes several major advances:

- Bachelier's (1900) random walk model of stock market fluctuations;
- Modigliani and Miller's (1958) cost of capital;
- Sharpe's (1964) capital asset pricing model;
- Samuelson (1965) and Fama's (1965; 1970) efficient market hypothesis;
- Black-Scholes' (1973) option pricing formula; and
- Heath-Jarrow-Morton's (1992) model of the term structure of interest rates.

First, I will review some of the latest realistic asset-pricing models being proposed. A summary of the debate on whether stock markets are deterministic or stochastic follows, together with a discussion of the statistical tools used to fit linear and nonlinear models. Last, a look at the recently proposed concept of scale relativity in theoretical physics might one day have several applications in finance.

2 Realistic Models

Early asset-pricing models, such as the ones mentioned in the introduction, make several simplifying assumptions. Many of these earlier assumptions are no longer necessary as new statistical methods become available and computing power grows exponentially. Market efficiency assumes investors have no memory in order to get independence of time series observations. Lillo (2004) demonstrates that long memory and market efficiency are compatible using tick-by-tick London Stock Exchange data for 1999 to 2002. Evolutionary game models also show great promise in finance (Ponzi and Aizawa, 2000; Friedmann, 2001). Evolutionary game models, and more complicated agent-based models, study the rich dynamics of financial markets observed in the real world.

2.1 Evolutionary Game Theory

Game Theory (von Neumann and Morgenstern, 1953; Williams, 1966; Davis, 1970) has been applied in several fields, including gambling, economics, international relations, evolutionary biology, political science, and military strategy.

Evolutionary Game Theory (Smith, 1981; Weibull, 1995; Hofbauer and Sigmund, 1998) studies the dynamics and equilibria of games played by populations of players. The strategies players employ in the games determine their interdependent payoff. In contrast to classical game theory, the players do not always act rationally when choosing their strategies. They exhibit *bounded rationality* (Simon, 1957, 1990; Sargent, 1993; Kahneman, 2003). Strategies evolve over time while players repeatedly play the game. More successful strategies are adopted by an increasing number of players, ie. they "invade" the population of current strategies, until even better ones are found.

The common methodology to study the evolutionary dynamics in games is through replicator equations (Hofbauer and Sigmund, 2003). The attractors (stable fixed points) of the equations are equivalent with evolutionarily stable strategies. Evolutionary game models can also be modeled using partial differential equations (Dorofeenko, 2003).

2.2 Technical and value strategies

Farmer (2002a,b) generalizes evolutionary market models by relaxing the equilibrium assumption. In his framework, markets are out-of-equilibrium and converge to equilibrium only asymptotically. Disequilibrium price formation uses a price adjustment process of the form

$$\frac{dP}{dt} = f\big(D(P)\big) \tag{1}$$

where P is the price at time t, D is the excess demand (demand-supply), and f is an increasing function known as the *market impact function.*

The rate of change of the price increases as there is excess demand for a security. Conversely, the rate of change of the price decreases when there is excess supply. Excess demand is a function of price so that different prices will change demand and supply for a security. Feedback between P and its derivative dP/dt leads to complex price dynamics.

Decision strategies, employed by investors and traders, can be classified into three broad groups. *Technical* or *chartist* strategies, of which *trend-following* is a special case, depend only on past prices. *Value* or *fundamental* strategies are based on a subjective assessment of value in relation to price. *Market making* involves another set of strategies to profit on the bid-ask spread while maintaining a minimal net position to reduce risk. Westerhoff and Reitz (2003) comment on market dynamics caused by chartists and fundamentalists using GARCH to model "switching regimes" in exchange rate markets.

Value investors buy when they believe an asset is undervalued and sell, or take a short position, when they see an asset as overvalued. As a result, value strategies induce negative short-term autocorrelations in price. Trend-following strategies, conversely, induce positive short-term autocorrelations since trend-following investors buy when the price of an asset has been recently rising and sell when the price of an asset has been declining. Real markets are populated by both value and trend-following investors resulting in empirically observed autocorrelations close to zero (Farmer, 2002a). Market makers also induce short-term endogenous market dynamics (Theodosopoulos and Badshah, 2004).

Induced autocorrelations on longer timescales display more complex behavior (Farmer, 2002a). Long-term nonequilibrium price dynamics tend to have second-order oscillatory terms which contribute to the nonlinear structure seen in real market data such as irregular cycles (Potter, 2000; Atanasova, 2003) and bursts of volatility (Engle, 1982; Shiller, 1997).

Farmer (2002a) studies the long-term evolution of markets as a dynamical system of money flows. Capital is allocated to various investment strategies as a function of previous performance. The resulting equations are known as the *Generalized Lotka-Volterra equations* (Brenig, 1988; Solomon, 1998), a standard model of population dynamics in biology. These equations can have stable or unstable dynamics, with fixed points, limit cycles or chaotic attractors.

2.3 Agent-based financial modeling

Evolutionary game theory studies the evolution of equilibria of relatively simple populations of competing strategies. Agent-based models (LeBaron, 2000; Farmer, 2002c; Farmer and Patelli, 2005a,b) capture more complex attributes of real financial markets that include, for example, learning and herding behaviors. Unfortunately, analytical results are hard to obtain for agent-based models due to their complexity.

Properties arising from the collective behavior of several agents are called *emergent properties* (Holland, 1998; Chen and Yeh, 2002). Markets can be composed of *heterogenous agents* where different classes of investors, eg. large institutions and retail investors, behave very differently (Jonsson and Keppo, 2002).

3 Deterministic or Stochastic ?

There has been a debate on whether stock markets are deterministic or stochastic ever since fractal geometry and chaos theory were first introduced to finance (Scheinkman and LeBaron, 1989; Peters, 1989, 1991a,b; Mandelbrot, 1997).

Are stock returns generated by deterministic nonlinear dynamical systems with sensitive dependence on initial conditions, also known as chaos (Chen, 1996)? Or do the returns follow a random walk? There isn't much difference as the two are both unpredictable. In fact, mathematicans are now studying deterministic nonlinear differential equations in a probabilistic way (Graham et al., 1996; Cerrai, 2001).

The real question is whether returns can be predicted (Fuksa, 1997, 2002) and if profits can be made using such forecasts (Felix, 2003). Ramsey (1996), highly critical of the predictive abilities of nonlinear models, raises an interesting question: if nonlinear models cannot forecast, what use are they? Peters (1991a, 1994, 1999a,b) proves that one way to make money using chaos theory and finance is to write several books on the topic.

3.1 Stochastic Models

Econometrics employs several stochastic models to analyze and predict the returns generating process:

- AR: Autoregressive (Kennedy, 2003);
- MA: Moving Average (Kennedy, 2003);
- ARMA: Autoregressive Moving Average (Box and Jenkins, 1970);
- ARIMA: Autoregressive Integrated Moving Average (Box and Jenkins, 1970);
- NARMA: Nonlinear Autoregressive Moving Average (Narendra and Mukhopadhyay, 1997; Lu and Chon, 2003);
- ARCH: Autoregressive Conditional Heteroscedasticity (Bollerslev et al., 1992);
- GARCH: Generalized Autoregressive Cond. Heteroscedasticity (Bollerslev et al., 1992).

These stochastic models look at previous prices (autoregressive), account for mean reversion (moving average), and attempt to identify the probability distribution from which random returns are drawn. The underlying distribution can be nonstationary (heteroscedasticity), ie. its parameters can change over time.

Stochastic Differential Equations (Oksendal, 2003), often used in finance, allow for randomness in some or all of the coefficients of a differential equation. Consider the deterministic population growth model

$$\frac{dN}{dt} = a(t)N(t), \qquad N(0) = N_0(constant) \tag{2}$$

where $N(t)$ is the population at time t, and $a(t)$ is the rate of growth at time t. Suppose $a(t)$ is not completely known, but equal to $r(t) + \xi(t)$ where $r(t)$ is known and deterministic. Exact behavior of random noise $\xi(t)$ is not known, except for its probability distribution. The original deterministic differential equation (2) then becomes a stochastic differential equation:

$$\frac{dN}{dt} = \Big(r(t) + \xi(t)\Big)N(t), \qquad N(0) = N_0(constant) \tag{3}$$

3.2 Nonlinear Models

The literature on nonlinear systems is extensive. Giannakis and Serpedin (2001) compiled a bibliography of 1410 articles, published between 1970 and 2000, on various topics related to nonlinear systems such as identification (ie. testing for nonlinearity), approximation, and prediction.

Nonlinear models are now commonly used in finance. For example, nonlinear models have been used to study:

- exchange rates (Higgins and Bera, 1992; Westerhoff and Reitz, 2003);
- European GNP data (Gatti and Gallegati, 1998);
- term structure of interest rates (Ahm and Gao, 1999; Chapman, 2000; Björk and Svensson, 2001);
- energy shocks to financial markets (Ciner, 2001);
- emerging markets (Taylor and Sarno, 2001; Iregui et al., 2002);
- Canadian money demand (Cushman, 2002); and

- firm value / performance vs. ownership structure, equity issues, corporate payout policy, corporate debt, corporate cash holdings, corporate investment decisions, board structure, and risk-taking behavior of financial institutions (Chen et al., 2004);

Teräsvirta et al. (1994) provide a good guide to nonlinear analysis of economic time series. Small and Tse (2003) look at the Dow Jones Industrial Average, London gold, and the USD/JPY exchange rate using correlation dimension (Grassberger and Procaccia, 1983a,b; Judd, 1992, 1994), nonlinear prediction error (Sugihara and May, 1990), and surrogate data analysis (Brock et al., 1991). Small and Tse (2003) conclude that there exists detectable deterministic nonlinearity, not captured by ARCH models, that can potentially be exploited for prediction.

3.3 Fitting Models

Functions in one or more variables can be approximated using *Taylor Series* (Weisstein, 2005)

$$f(x) \approx a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + a_3(x - x_0)^3 + ... \tag{4}$$

around any given point x_0. A univariate linear model would be $f(x) = a_0 + a_1(x - x_0)$ while a nonlinear model would include some of the higher-order terms $a_2(x - x_0)^2$, $a_3(x - x_0)^3$, ...

For a function of two variables, (4) becomes

$$\begin{aligned} f(x, y) \approx a_{00} & \\ & + a_{10}(x - x_0) + a_{01}(y - y_0) \\ & + a_{20}(x - x_0)^2 + a_{11}(x - x_0)(y - y_0) + a_{02}(y - y_0)^2 \\ & + a_{30}(x - x_0)^3 + a_{21}(x - x_0)^2(y - y_0) + a_{12}(x - x_0)(y - y_0)^2 + a_{03}(y - y_0)^3 \\ & + ... \end{aligned} \tag{5}$$

A linear model would now include the a_{00}, a_{10}, and a_{01} terms. Stepwise Regression (Efroymson, 1960; Hocking, 1977; Jennrich, 1977, 1995) can be used to select a subset of variables, or terms in this case, which best fits some data. Efroymson's algorithm, known to converge in a finite number of steps (Miller, 1996), has been improved many times (Breaux, 1968; Miller, 1984, 1992a,b). McIntyre et al. (1983) evaluate the statistical significance of models obtained using stepwise regression. To increase robustness to outliers, Agostinelli (2002) introduces a weighted likelihood F-test to replace the standard F-test used at each step of the algorithm. The modern version (Miller, 2002) of Efroymson's algorithm uses QR-decomposition (Anderson et al., 1999; Galassi et al., 2005) of the regression design matrix X so that

$$X = QR = \left(Q_1 \middle| Q_2 \middle| \cdots \middle| Q_k \right) \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1k} \\ & r_{22} & \cdots & r_{2k} \\ & & \ddots & \vdots \\ & & & r_{kk} \end{pmatrix} \tag{6}$$

where Q is an orthonormal basis and R is an upper-triangular matrix.

Nonlinear Regression (Motulsky and Ransnas, 1987; Seber, 1989), on the other hand, fits models with arbitrary functional forms $f(x_1, x_2, ..., x_k)$ where f is no longer an independent linear combination of explanatory variables. For example,

$$y \approx \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k \tag{7}$$

is a linear regression model with parameters $\beta_0, \beta_1, \beta_2, ..., \beta_k$, while

$$y \approx \beta_0 + \beta_1 e^{\beta_2 x} \tag{8}$$

and

$$y \approx \beta_0 + \beta_1 \left(\frac{1}{x_1} + \frac{1}{x_2} \right) + \frac{\beta_2}{x_1 x_2} \tag{9}$$

are nonlinear regression models with parameters β_0, β_1, and β_2. Some nonlinear models can be made linear via a suitable transformation, eg.

$$y \approx e^{\beta_0+\beta_1 x} \tag{10}$$

can be transformed to linear model $\beta_0 + \beta_1 x$ by taking logarithms.

There is no analytical solution in nonlinear regression like the solution

$$\boldsymbol{\beta} = \left(\mathbf{X}^{\mathbf{T}}\mathbf{X}\right)^{-1} \mathbf{X}^{\mathbf{T}}\mathbf{y} \tag{11}$$

to the normal equations in linear regression. Nonlinear regression parameters must be computed using iterative numerical algorithms to minimize least-squares or some other goodness-of-fit measure (Seber, 1989; Galassi et al., 2005; Olinsky et al., 2004). Greene (2003) has an excellent chapter on nonlinear regression while Kennedy (2003) avoids the topic.

4 Scale Relativity

Physicists are increasingly contributing to finance (Farmer, 1999; Dragulescu, 2003) with interesting applications coming from turbulence (Lux, 2001), statistical mechanics (Ingber, 1984, 1996; Voit and Lourie, 2003), and other physics theories. Scale relativity (Nottale, 1992, 1993a), a recent theory in theoretical physics, might someday bring numerous insights to finance, which has several examples of scale:

- investment horizons (intra-day, daily, weekly, monthly, quarterly, yearly);
- company size (small cap, medium cap, large cap);
- investor class (retail investors, hedge funds, mutual funds, financial institutions); and
- "economies of scale".

Einstein's theory of relativity applies to the states of position (origin and orientation of axes) and of motion of the system of coordinates (velocity, acceleration). These states are relative, they are never definable in an absolute way. The state of any system can be defined only in comparison with another system.

Nottale suggested that the observation scale, the resolution at which a system is observed or experimented with, should also be considered as a characteristic of the state of reference systems. The scale of a system can be defined only in a relative way: only scale ratios have a physical meaning, never absolute scales. This led Nottale to extend the principle of relativity to the scale of systems. In his new approach, resolution is interpreted not only as a property of the measuring device and of the measured system, but more generally as a property that is intrinsic to the geometry of space-time itself. Space-time is now considered to be fractal.

4.1 Quantum Mechanics and Fractal Geometry

Scale relativity generalizes the geometry of space-time, currently twice-differentiable, by abandoning the hypothesis of differentiability of space-time coordinates. More abstract spaces, of which differentiable ones are a subset, must be considered. Nottale (1993b) replaces the Newtonian time derivative dx/dt, which now diverges as $dt \to 0$, with the differential operator

$$dX = dx + d\xi \tag{12}$$

where dx represents the "classical part" and $d\xi$ the "fractal" part. Nottale then derives the famous Schrödinger equation of quantum mechanics as the consequence of the geodesics on a fractal space-time. This landmark theoretical physics result links the stochastic nature of quantum mechanics to the fractal nature of space-time at small scales. Feynman indeed believed typical paths of quantum mechanics to be continuous but non-differentiable (Feynman and Hibbs, 1965). However, the term "fractal" was not coined until several years later by Mandelbrot (1975, 1985).

4.2 Predictability and Scale

Exact measurement and prediction are possible at large scales. For example, a baseball in flight follows a well-known parabolic equation. However at small scales, eg. electrons in the baseball's leather, measurement and prediction accuracy is limited according to Heisenberg's *uncertainty principle*. Similarly, daily individual stock prices are rather unpredictable while yearly changes of stock indexes and economic indicators are much easier to predict.

Where does the transition between non-predictability and predictability occur? Célérier and Nottale (2004) answer this question by looking at the transition from non-differentiability (fractal scales) to differentiability (classical scales) for the laws of motion in physics. The relationship between predictability and scale in finance remains an open question.

5 Conclusion

Finance achieved significant advances in the twentieth century. Progress towards better models of financial markets continues.

Evolutionary game models, and the more complicated agent-based models, show great promise. Statistical methods and computing power continuously advance in parallel with finance. New models better approximate reality by dropping previous simplifying assumptions that are no longer necessary. There is support for both deterministic and stochastic models but both can be unpredictable. Linear and nonlinear regression algorithms are mature with well-known statistical properties. Scale relativity should bring major insights to finance in the next few years as finance is filled with examples of scales.

This literature survey will form the basis of my MBA thesis next year. Additional knowledge uncovered during this review of the latest research in finance will certainly be useful.

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